

# Composite point FFT

Composite no  $\rightarrow$  more than two factors.

Example 3 -

$$6 = 2 \times 3$$

$$N = P_1 N_1$$

$$N = P_1 N_1$$

$P_1 \rightarrow$  no. of subsequence

$N_1 \rightarrow$  no. of elements in subsequence.

for  $N=6 = 2 \cdot 3$ .

Subsequences  $x(0) \ x(3) \ \text{--- (1)}$

$x(1) \ x(4) \ \text{--- (2)}$

$x(2) \ x(5) \ \text{--- (3)}$

for  $N=6 = 2 \cdot 3$ .

$x(0), x(2), x(4)$

$x(1), x(3), x(5)$

for  $N=6 = 3 \cdot 2$

$x(0)$	$x(1)$	$x(2)$
$x(3)$	$x(4)$	$x(5)$

$x(3r) \quad x(3r+1) \quad x(3r+2)$   
 $r=0, 1, \dots, (N_1-1)$

for  $N=6 = 2 \cdot 3$ .

$x(0)$	$x(1)$
$x(2)$	$x(3)$
$x(4)$	$x(5)$

$x(2r) \quad x(2r+1)$

$r=0, 1, \dots, (N_1-1)$

for  $N=6 = 3 \cdot 2$

$x(0)$	$x(2)$	$x(4)$
$x(1)$	$x(3)$	$x(5)$
$x(6)$	$x(7)$	$x(8)$

$x(3r) \quad x(3r+1) \quad x(3r+2)$   
 $r=0, 1, \dots, (N_1-1)$

$$X(K) = \sum_{r=0}^{N_1-1} x(3r) \omega_N^{3rK} + \sum_{r=0}^{N_1-1} x(3r+1) \omega_N^{(3r+1)K} + \sum_{r=0}^{N_1-1} x(3r+2) \omega_N^{(3r+2)K}$$

for  $N \geq 9$

$$X(K) = \sum_{r=0}^{N_1-1} x(3r) \omega_N^{3rK} + \sum_{r=0}^{N_1-1} x(3r+1) \omega_N^{(3r+1)K} + \sum_{r=0}^{N_1-1} x(3r+2) \omega_N^{(3r+2)K}$$

general Composite FFT for  $N = P_1 N_1$

$$X(K) = \sum_{r=0}^{N_1-1} x(P_1 r) \omega_N^{P_1 r K} + \sum_{r=0}^{N_1-1} x(P_1 r+1) \omega_N^{(P_1 r+1)K} + \dots + \sum_{r=0}^{N_1-1} x(r P_1 + P_1 - 1) \omega_N^{(r P_1 + P_1 - 1)K}$$

for  $N=6$

①  $6 = 2 \times 3$   $P_1=2$   $N_1=3$

$$X(k) = \sum_{r=0}^2 x(2r) \omega_6^{2rk} + \sum_{r=0}^2 x(2r+1) \omega_6^{(2r+1)k}$$

$$X_1(k) = X_2(k)$$

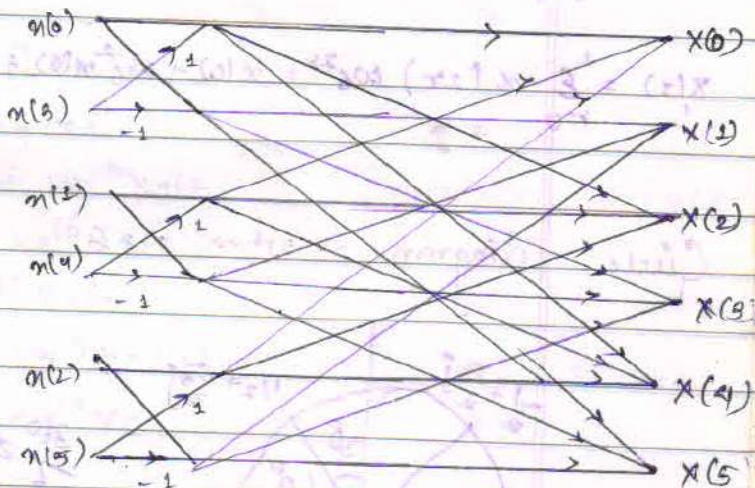
②  $N=6=3 \times 2$   $P_1=3$   $N_1=2$

$$X(k) = \sum_{r=0}^1 x(3r) \omega_6^{3rk} + \sum_{r=0}^1 x(3r+1) \omega_6^{(3r+1)k}$$

$$+ \sum_{r=0}^1 x(3r+2) \omega_6^{k(3r+2)}$$

$k=0, 1, 2, \dots, 5$

$$X(k) = \sum_{r=0}^1 x$$



$$X(1) = X_1(0) + \omega_6^1 X_2(1) + \omega_6^2 X_3(2) \text{---(b)}$$

$$X(2) = X_1(0) + \omega_6^2 X_2(0) + \omega_6^4 X_3(0) \text{---(c)}$$

$$X(3) = X_1(1) + \omega_6^3 X_2(1) + \omega_6^6 X_3(1) \text{---(d)}$$

$$X(4) = X_1(0) + \omega_6^4 X_2(0) + \omega_6^8 X_3(0) \text{---(e)}$$

$$X(5) = X_1(1) + \omega_6^5 X_2(1) + \omega_6^0 X_3(1) \text{---(f)}$$

Three summation representation of 2 pt DFT of seq  $x(3r)$ ,  $x(3r+1)$  &  $x(3r+2)$

$$X(k) = X_1(k) + \omega_6^k X_2(k) + \omega_6^{2k} X_3(k) \text{---(1)}$$

$$X(k) = \sum_{r=0}^2 x(2r) \omega_6^{2rk} + \sum_{r=0}^2 x(2r+1) \omega_6^{(2r+1)k}$$

$$= \sum_{r=0}^1 x(2r) \omega_6^{2rk} + \omega_6^k \sum_{r=0}^1 x(2r+1) \omega_6^{2rk}$$

$k=0, 1, 2, \dots, 5$   $k=0, 1$   $k=0, 1$

3 pt DFT  $\downarrow$   $\downarrow$  2 pt DFT

put  $k=0$  in eq. (1)

$$X(k) = X_1(k) + \omega_6^k X_2(k)$$

$$X(0) = X_1(0) + \omega_6^0 X_2(0) + \omega_6^0 X_3(0)$$

$$X(0) = X_1(0) + X_2(0) + X_3(0) \text{---(a)}$$

Put  $K=0$  in equation (11)

we have,

$$X_m(0) = X_{M-1,2}(0) + W_N^0 X_{M-1,2}(0)$$

— (a)

for  $K=1$

$$X_m(1) = X_{M-1,2}(1) + W_N^1 X_{M-1,2}(1)$$

— (b)

for  $K=2$

$$X_m(2) = X_{M-1,2}(2) + W_N^2 X_{M-1,2}(2)$$

— (c)

for  $K=3$

$$X_m(3) = X_{M-1,2}(3) + W_N^3 X_{M-1,2}(3)$$

— (d)

for  $K=4$

$$X_m(4) = X_{M-1,2}(4) + W_N^4 X_{M-1,2}(4)$$

— (e)

$$= X_{M-1,2}(0) + W_N^4 X_{M-1,2}(0)$$

Since it is periodic by  $N/2$

$$\therefore 4=0$$

$\therefore \frac{N}{2}$  pt DFT

6 2 0 4  $\Rightarrow$  4 pt DFT

periodicity = 4.

for  $K=5$

$$X_m(5) = X_{M-1,2}(5) + W_N^5 X_{M-1,2}(5)$$

$$= X_{M-1,2}(1) + W_N^5 X_{M-1,2}(1)$$

— (f)

for  $K=6$

$$X_m(6) = X_{M-1,2}(6) + W_N^6 X_{M-1,2}(6)$$

$$= X_{M-1,2}(2) + W_N^6 X_{M-1,2}(2)$$

$$= X_{M-1,2}(2) + W_N^6 X_{M-1,2}(2)$$

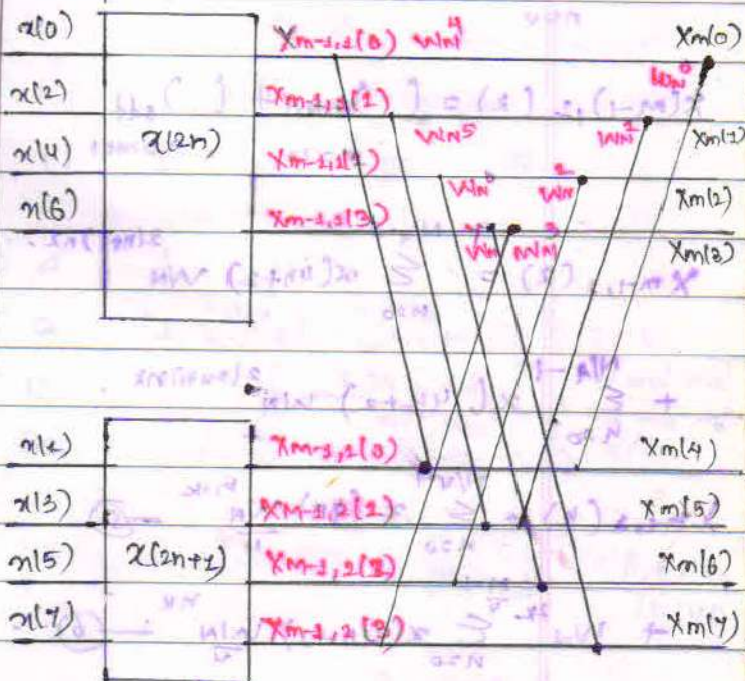
— (g)

for  $K=7$

$$X_m(7) = X_{M-1,2}(7) + W_N^7 X_{M-1,2}(7)$$

$$= X_{M-1,2}(3) + W_N^7 X_{M-1,2}(3)$$

— (h)



Butterfly Structure :-

further even pt DFT  $x(2n)$  and odd point DFT  $x(2n+1)$  are broken up in even and odd point DFT's.

ie (11) equation (11) & (10) are further divided into even & odd point DFT's.

In radix 2 smaller DFT will be the point 2 further N/2 DFT break into combination of 2 N/4

point DFT. It's decomposition of radix 2 DIT FFT starts from eq. (3) & (4).

from eq. (3)

$$X_{M-1,1}(k) = \sum_{n=0}^{N/2-1} x(2n) W_N^{nk}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_N^{2nk}$$

$$X_{M-1,2}(k) = \left( \sum_{n=0}^{N/2-1} x(2n) W_N^{nk} \right)_{\text{even}} + \left( \sum_{n=0}^{N/2-1} x(2n+1) W_N^{nk} \right)_{\text{odd}}$$

$$X_{M-1,1}(k) = \sum_{n=0}^{N/4-1} x(4n+2) W_N^{2(n+1)nk}$$

$$+ \sum_{n=0}^{N/4-1} x(4n+1) W_N^{2(2n+1)nk}$$

$$X_{M-1,2}(k) = \sum_{n=0}^{N/4-1} x(4n) W_N^{nk} \quad \text{--- (7)}$$

$$+ W_N^{nk} \sum_{n=0}^{N/4-1} x(4n+2) W_N^{nk} \quad \text{--- (8)}$$

for  $k=0, 1, \dots, (N/2-1)$

Eq. (7) represents N/4 pt DFT of seq.  $x(4n)$

Eq. (8) represents N/4 pt DFT of seq.  $x(4n+2)$

$$X_{M-1,1}(k) = X_{M-1,1}(k) + W_N^{nk} X_{M-1,2}(k)$$

$$X_{M-1,2}(k) = \sum_{n=0}^{N/2-1} x(2n+1) W_N^{nk}$$

$$X_{M-1,2}(k) = \sum_{n=0}^{N/4-1} x(4n+1) W_N^{nk}$$

Similarly for eq. (4)

$$X_{M-1,2}(k) = \sum_{n=0}^{N/2-1} x(2n+1) W_N^{nk}$$

$$= \left( \sum_{n=0}^{N/2-1} x(2n) W_N^{nk} \right)_{\text{even}} + \left( \sum_{n=0}^{N/2-1} x(2n+1) W_N^{nk} \right)_{\text{odd}}$$

$$X_{M-1,2}(k) = X_{M-1,3}(k) + W_N^{nk} X_{M-1,4}(k)$$

$$X_{M-1,3}(k) = \sum_{n=0}^{N/4-1} x(4n+1) W_N^{nk} \quad \text{--- (9)}$$

$$X_{M-1,4}(k) = \sum_{n=0}^{N/4-1} x(4n+3) W_N^{nk} \quad \text{--- (10)}$$

for  $N=8$  eq. (5) and (6) represents N/2 point DFT which is comb of two

N/4 pt DFT. Eq. (7), (8), (9), (10) rep. N/4 pt DFT

of 2 eqs.  $x(4n)$ ,  $x(4n+2)$ ,  $x(4n+1)$ ,  $x(4n+3)$ , for these eqs.  $k=0, 1, \dots, (N/4-1)$

for  $N=8$  from eq. (5) & (6)

for  $k=0$

$$X_{M-1,1}(0) = X_{M-1,2}(0) + W_N^{0k} X_{M-1,3}(0) \quad \text{--- (11)}$$

for  $k=1$

$$X_{M-1,1}(1) = X_{M-1,2}(1) + W_N^{1k} X_{M-1,3}(1)$$

for  $k=2$

$$X_{M-1,1}(2) = X_{M-1,2}(2) + W_N^{2k} X_{M-1,3}(2) \quad \text{--- (12)}$$

for  $K=3$ .

$$X_{M-1,1}(3) = X_{M-2,1}(2) + W_N^3 X_{M-2,2}(2) \quad \text{--- (d)}$$

In eqn (3)

for  $K=0$

$$X_{M-1,2}(0) = X_{M-2,3}(0) + W_N^0 X_{M-2,4}(0) \quad \text{--- (m)}$$

for  $K=1$

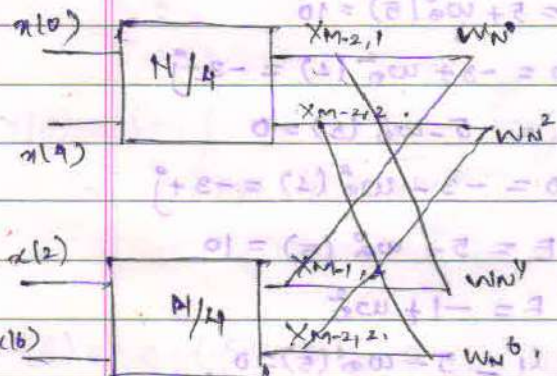
$$X_{M-1,2}(1) = X_{M-2,3}(1) + W_N^2 X_{M-2,4}(1) \quad \text{--- (n)}$$

for  $K=2$

$$X_{M-1,2}(2) = X_{M-2,3}(2) + W_N^4 X_{M-2,4}(2) \quad \text{--- (o)}$$

for  $K=3$ .

$$X_{M-1,2}(3) = X_{M-2,3}(3) + W_N^6 X_{M-2,4}(3) \quad \text{--- (p)}$$



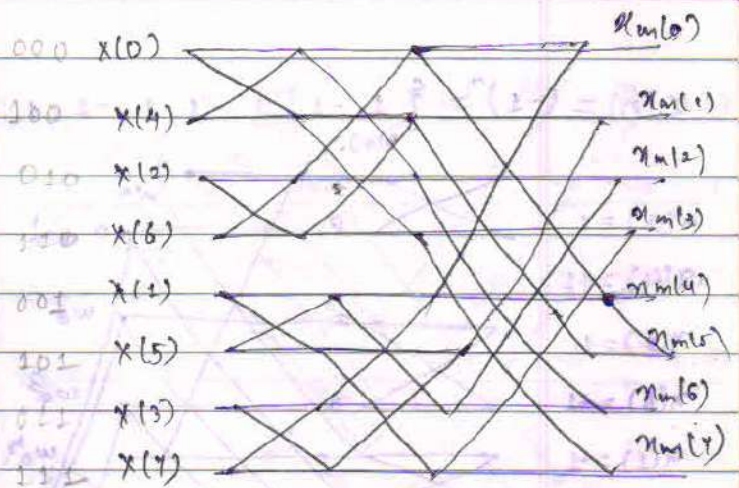
$x(0)$   $X_{M-2,1}(0)$

$x(4)$   $X_{M-2,1}(1)$

$x(2)$   $X_{M-2,2}(0)$

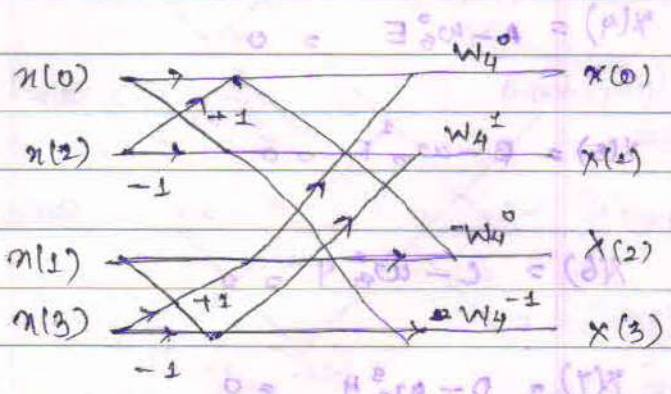
$x(6)$   $X_{M-2,2}(1)$

$$\begin{aligned} X(0) &= D \cdot W_N^0 \cdot 1 \\ X(1) &= E \cdot W_N^0 \cdot 1 \\ X(2) &= F \cdot W_N^0 \cdot 1 \\ X(3) &= G \cdot W_N^0 \cdot 1 \\ X(4) &= H \cdot W_N^0 \cdot 1 \end{aligned}$$



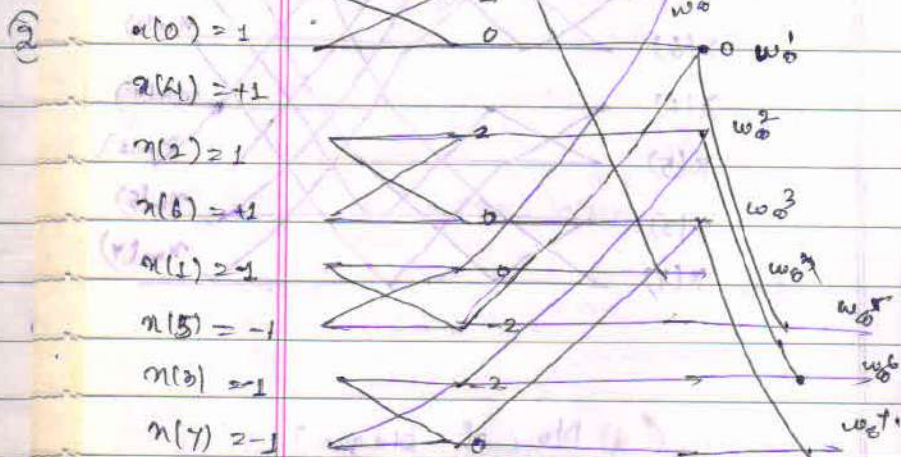
No. of stages

N	I	II	III	IV
2	1	$W_N^0, W_N^4$	$W_N^0, W_N^4$	$W_N^0, W_N^4$
4	1	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$
8	1	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$
16	1	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$
32	1	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$
64	1	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$	$W_N^0, W_N^2, W_N^4, W_N^6$



Q. Determine 8-point DFT of seq.  
 $x(n) = (-1)^n$  using Radix 2  
 decimation in Time FFT algorithm.

1)  $x(n) = (-1)^n = \{ \underset{x(0)}{1}, -1, \underset{x(1)}{1}, -1, \underset{x(2)}{1}, -1, \underset{x(3)}{1}, -1 \}$



$$X(0) = A + w_8^0 E = 4 + w_8^0 - 4$$

$$X(1) = B + w_8^1 F = 0 + w_8^1 + 0$$

$$X(2) = C + w_8^2 G = 0$$

$$X(3) = D + w_8^3 H = 4 - w_8^3(-4) = 8$$

$$X(4) = A - w_8^0 E = 0$$

$$X(5) = B - w_8^1 F = 0$$

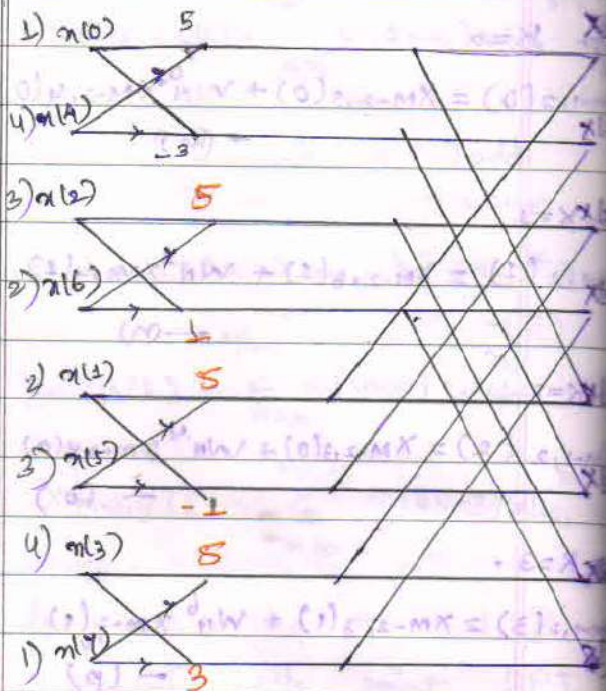
$$X(6) = C - w_8^2 G = 0$$

$$X(7) = D - w_8^3 H = 0$$

Q. Determine 8-pt DFT of  
 Sequence

$$x(n) = \{ 1, 2, 3, 4, 4, 3, 2, 1 \}$$

using Radix-2 DITFFT



$$A = 5 + w_8^0(5) = 10$$

$$B = -3 + w_8^1(1) = -3 - j$$

$$C = 5 - w_8^2(5) = 0$$

$$D = -3 - w_8^3(1) = -3 + j$$

$$E = 5 + w_8^4(5) = 10$$

$$F = -1 + w_8^5$$

$$G = 5 - w_8^6(5) = 0$$

$$H = -1 - w_8^7 = -1 + 9j$$

$$X(0) = A + w_8^0 E = 20$$

$$X(1) = B + w_8^1 F$$

$$X(2) = C + w_8^2 G = 0$$

$$X(3) = D + w_8^3 H$$

$$X(4) = A - w_8^4 E = 0$$

$$X(5) = B - w_8^5 F$$

$$X(6) = C - w_8^6 G = 0$$

$$X(7) = D - w_8^7 H$$

$$X(0) = 5 + W_8^0(5) + W_8^8 [5 + W_8^8(5)]$$

$$= 10 + 10$$

$$= 20$$

$$X(1) = [-3 + W_8^2(1)] + W_8^4 [-1 + W_8^6(3)]$$

$$= -3 - j + W_8^4 [-1 - 3j]$$

$$= -3 - j + (0.707 - 0.707j)(-1 - 3j)$$

$$= -5.828 - j \cdot 2.414$$

$$X(2) = 0 + (-j) \cdot 0 = 0$$

$$X(3) = (-3 + j) - (-0.707 - j \cdot 0.707) \cdot (-1 + 3j)$$

$$= -0.172 - 0.414j$$

$$X(4) = 10 - 10 = 0$$

$$X(5) = (-3 - j) - (0.707 - 0.707j)(-1 - 3j)$$

$$= -0.172 + 0.414j$$

$$X(6) = 0 - (-j) \cdot 0 = 0$$

$$X(7) = (-3 + j) - (-0.707 - 0.707j)(-1 + 3j)$$

$$= 5.828 + 2.414j$$

$$X(k) = \{ 20, -5.828 - 2.414j, 0, 0.172 - 0.414j, 0, 0.172 + 0.414j, 0, 5.828 + 2.414j \}$$

→ ans

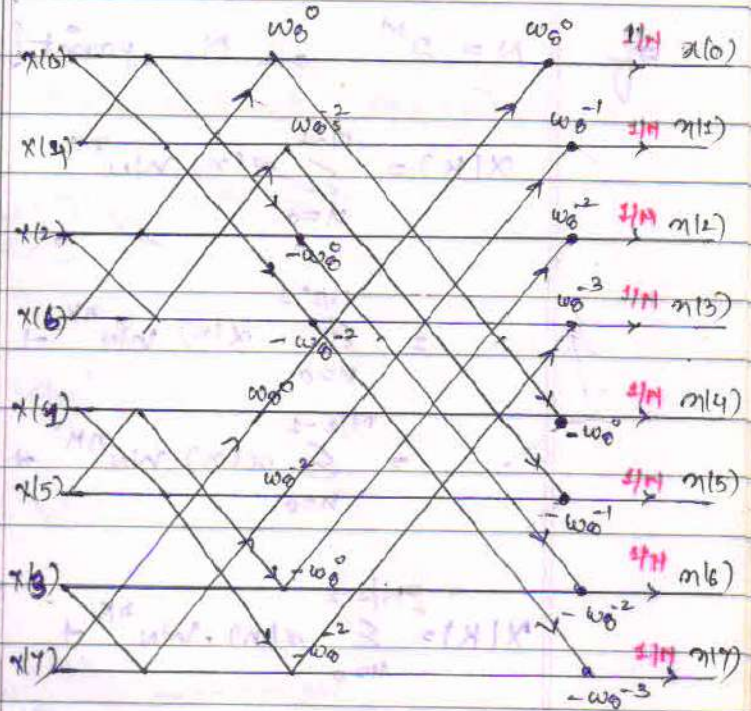
FOR IDFT :-

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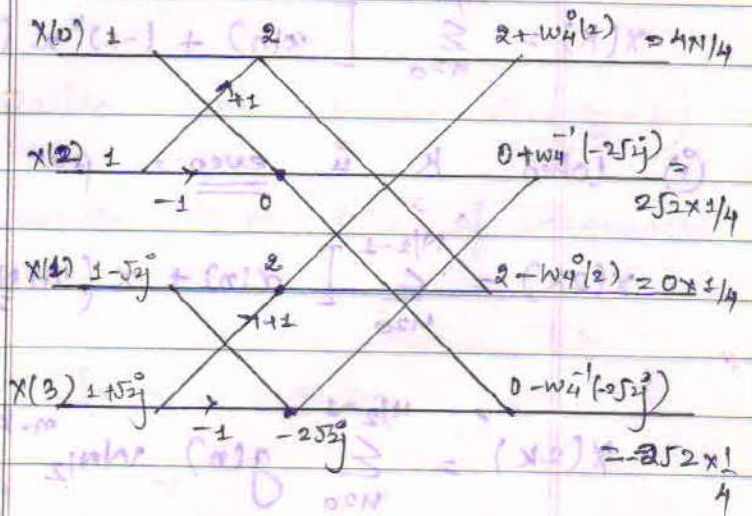
$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot W_N^{-kn}$$



Q. Determine 4 pt IDFT of

$$X(k) = \{ 1, 1 + \sqrt{2}j, 1, 1 + \sqrt{2}j \}$$

using Radix-2.



$$\therefore x(n) = \left\{ 1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right\}$$

# → Radix - 2 Decimation In FREQUENCY

## Algorithm :-

If  $N = 2^M$ ,  $N$  point DFT of seq.  $x(n)$  is -

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK}, \quad K=0, 1, \dots, (N-1) \quad \text{--- (1)}$$

$$= \sum_{n=0}^{N/2-1} x(n) W_N^{nK} + \sum_{n=0}^{N/2-1} x(n) W_N^{nK}, \quad K=0, 1, \dots, (N-1)$$

$$= \sum_{n=0}^{N/2-1} x(n) W_N^{nK} + \sum_{n=0}^{N/2-1} x\left(n + \frac{N}{2}\right) W_N^{(n + \frac{N}{2})K}, \quad K=0, 1, \dots, (N-1)$$

$$X(K) = \sum_{n=0}^{N/2-1} x(n) \cdot W_N^{nK} + \sum_{n=0}^{N/2-1} x\left(n + \frac{N}{2}\right) W_N^{nK} \cdot W_N^{N/2K}$$

Since,

$$W_N^N = 1, \quad W_N^{N/2} = -1 \quad (\text{half symmetry})$$

$$\therefore X(K) = \sum_{n=0}^{N/2-1} x(n) W_N^{nK} + \sum_{n=0}^{N/2-1} x\left(n + \frac{N}{2}\right) W_N^{nK} (-1)^K$$

$$X(K) = \sum_{n=0}^{N/2-1} \left[ x(n) + (-1)^K x\left(n + \frac{N}{2}\right) \right] W_N^{nK} \quad \text{--- (1)}$$

(i) When  $K$  is even, put  $K = 2k$  in eq. (1)

$$X(2k) = \sum_{n=0}^{N/2-1} \left[ x(n) + x\left(n + \frac{N}{2}\right) \right] W_N^{n \cdot 2k}$$

$$X(2k) = \sum_{n=0}^{N/2-1} g(n) W_{N/2}^{n \cdot k} \quad \text{--- (11)}$$

where  $g(n) = \left[ x(n) + x\left(n + \frac{N}{2}\right) \right] \quad \text{--- (12)}$



Equation (iii) represent  $N/2$  point DFT of sequence  $g(n)$ , for this equation  $k$  varies from  $0, 1, 2, \dots, (\frac{N}{2}-1)$ .

(ii) When  $k$  is ODD, put  $k = (2K+1)$  in eq. (ii)

$$X(2K+1) = \sum_{n=0}^{N/2-1} [x(n) - x(n + \frac{N}{2})] W_N^{(2K+1)n}$$

$$X(2K+1) = \sum_{n=0}^{N/2-1} h(n) W_N^{2Kn} \cdot W_N^n$$

$$X(2K+1) = \sum_{n=0}^{N/2-1} [h(n) \cdot W_N^n] \cdot W_N^{2Kn}$$

$$X(2K+1) = \sum_{n=0}^{N/2-1} h(n) W_N^n \cdot W_N^{nK} \quad \text{--- (iv)}$$

where  $h(n) = x(n) - x(n + \frac{N}{2})$  --- (v)

Equation (v) represents  $\frac{N}{2}$  point DFT of the

sequence  $h(n) W_N^n$ .

for this equation  $k$  varies from  $0, 1, 2, \dots, (\frac{N}{2}-1)$

for  $N=8$  eq. (iii) & (v) represents 4 pt DFT of sequences  $g(n)$  and  $h(n) W_N^n$ .

for these equations  $k$  varies from 0 to 3.

$$g(0) = x(0) + x(4) \quad \text{--- (a)}$$

$$g(1) = x(1) + x(5) \quad \text{--- (b)}$$

$$g(2) = x(2) + x(6) \quad \text{--- (c)}$$

$$g(3) = x(3) + x(7) \quad \text{--- (d)}$$

} Even point DFT.

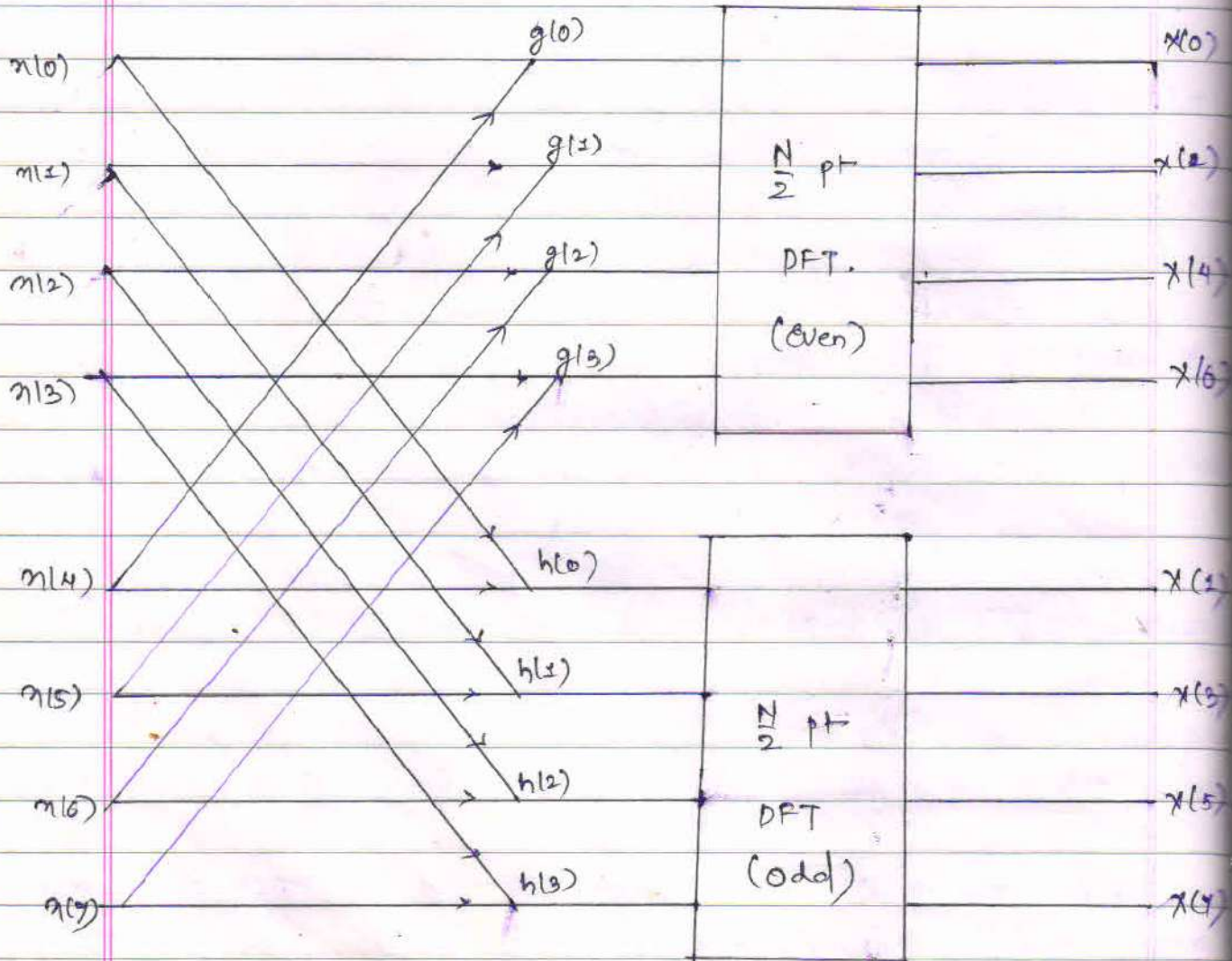
Also,  $h(0) = x(0) - x(4) \text{ --- (e)}$

$h(1) = x(1) - x(5) \text{ --- (f)}$

$h(2) = x(2) - x(6) \text{ --- (g)}$

$h(3) = x(3) - x(7) \text{ --- (h)}$

$x \cdot W_N^n = \text{Odd point DFT}$



In radix-2 smaller DFT will be the point 2, so 2nd decomposition starts from (III) or (V).

from eq. (5)

$$x(2k) = \sum_{n=0}^{N/2-1} g(n) W_{N/2}^{nk}, \quad k=0, 1, 2, \dots, \left(\frac{N}{2}-1\right)$$

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Page (N/4) .....

$$X(2k) = \sum_{n=0}^{N/4-1} g(n) W_N^{2nk} + \sum_{n=N/4}^{N/2-1} g(n) W_N^{2nk}$$

$$= \sum_{n=0}^{N/4-1} g(n) W_N^{2nk} + \sum_{n=0}^{N/4-1} g\left(n + \frac{N}{4}\right) W_N^{2\left(n + \frac{N}{4}\right)k}$$

$$= \sum_{n=0}^{N/4-1} \left[ g(n) W_N^{2nk} + g\left(n + \frac{N}{4}\right) W_N^{2nk} \cdot W_N^{2N/4 k} \right]$$

$$= \sum_{n=0}^{N/4-1} \left[ g(n) W_N^{2nk} + g\left(n + \frac{N}{4}\right) W_N^{2nk} \cdot W_N^{N/2 k} \right]$$

$$\therefore X(2k) = \sum_{n=0}^{N/4-1} \left[ g(n) + g\left(n + \frac{N}{4}\right) (-1)^k \right] W_N^{2nk} \quad \text{--- (7)}$$

for  $k$  is even  $k=2k$  in eq. (7)

$$X(4k) = \sum_{n=0}^{N/4-1} \left[ g(n) + g\left(n + \frac{N}{4}\right) \right] W_N^{4nk}$$

$$X(4k) = \sum_{n=0}^{N/4-1} \left[ g(n) + g\left(n + \frac{N}{4}\right) \right] W_{N/4}^{nk}$$

$$X(4k) = \sum_{n=0}^{N/4-1} \left[ A(n) \cdot W_{N/4}^{nk} \right] \quad \text{--- (VIII)}$$

where  $A(n) = g(n) + g\left(n + \frac{N}{4}\right)$  --- (IX)

for  $k$  is odd put  $k=(2k+1)$  in equation (VII)

$$X(4k+2) = \sum_{n=0}^{N/4-1} \left[ g(n) - g\left(n + \frac{N}{4}\right) \right] W_N^{2n(2k+1)}$$

$$= \sum_{n=0}^{N/4-1} \left[ g(n) - g\left(n + \frac{N}{4}\right) \right] W_N^{4nk} \cdot W_N^{2n}$$

$$X(4k+2) = \sum_{n=0}^{N/4} B(n) W_N^{2n} W_N^{n/4} \quad \text{--- (x)}$$

where  $B(n) = g(n) - g(n + \frac{N}{4})$  --- (x1)

for  $N=8$  eqo (viii) and (x) represents 2 pt DFT of the sequence  $A(n)$  and  $B(n) \cdot W_N^{2n}$ .

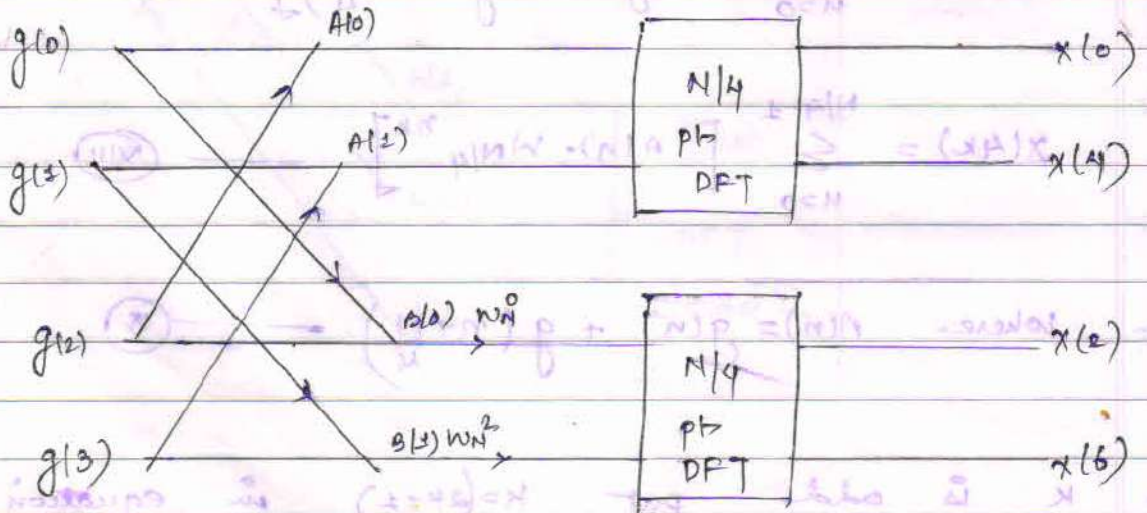
for these equations  $k$  varies as 0 and 1.

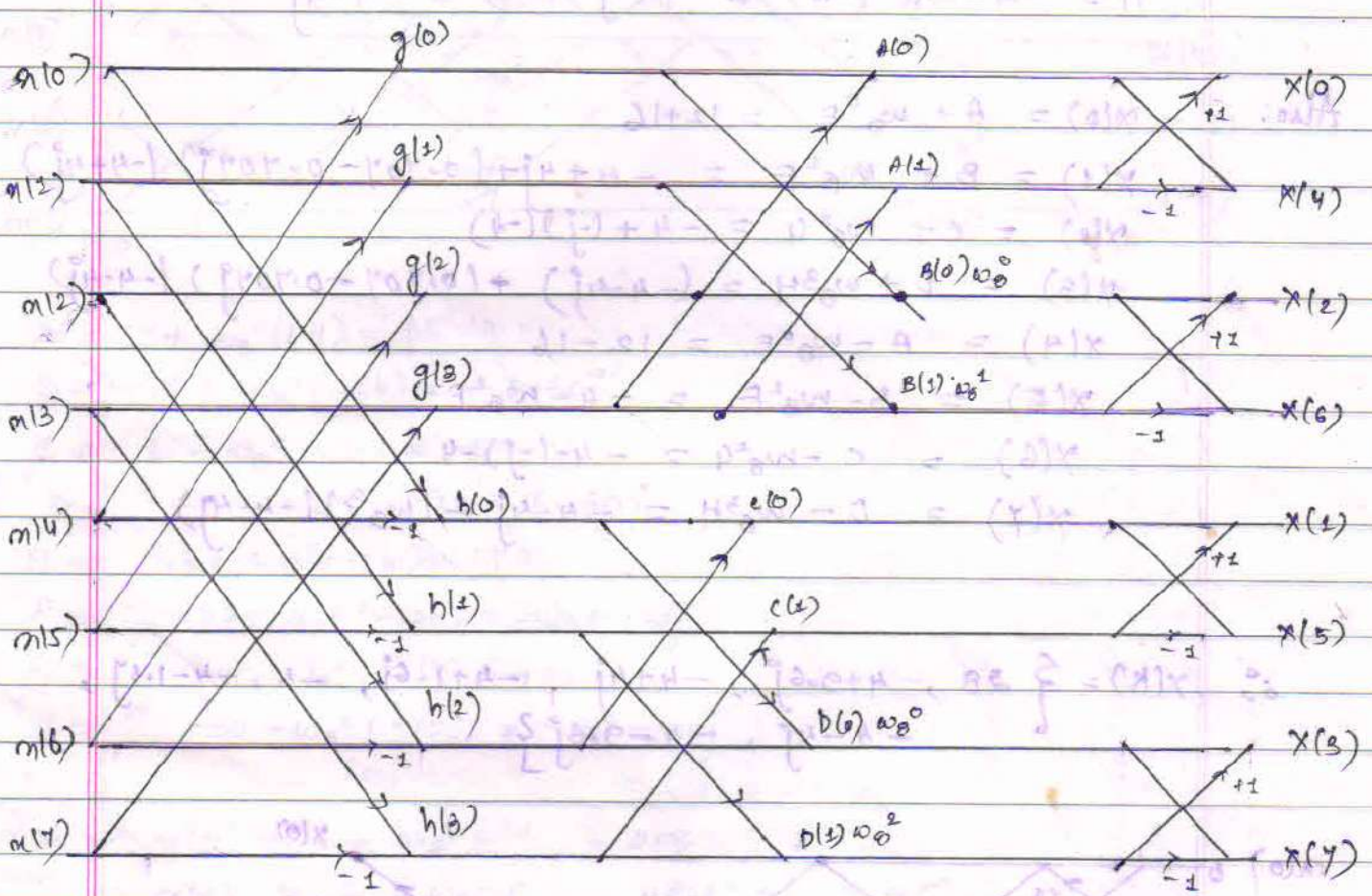
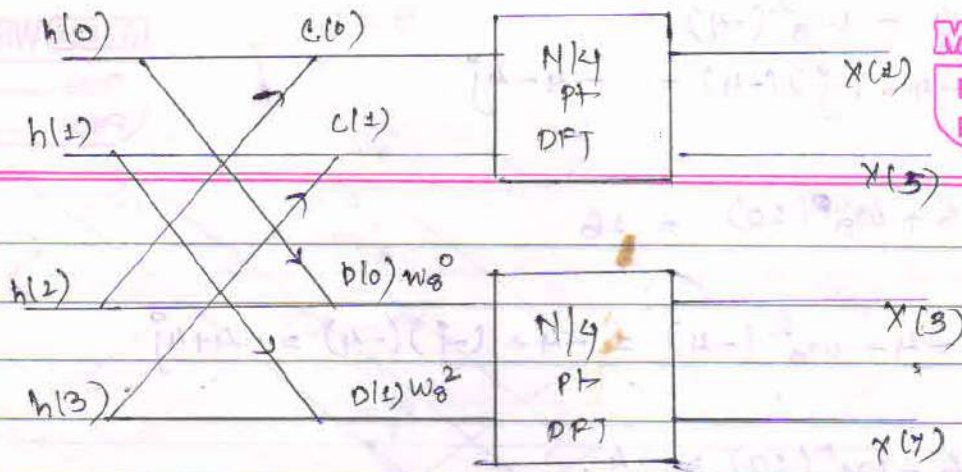
$$A(0) = g(0) + g(\frac{N}{4}) = g(0) + g(2) \quad \text{--- (i)}$$

$$A(1) = g(1) + g(3) \quad \text{--- (j)}$$

$$B(0) = g(0) - g(2) \quad \text{--- (k)}$$

$$B(1) = g(1) - g(3) \quad \text{--- (l)}$$





Q.  $x(n) = \{0, 1, 2, 3, 4, 5, 6, 7\}$ ;  $N = 8$

$A = 4 + 8\omega_8^0 = 12$

$B = -4 + \omega_8^2(-4)$   
 $= -4 + (-j)(-4) = -4 + 4j$

$C = 8 - \omega_8^0(8) = -4$

$$D = -4 - \omega_8^2(-4) = -4 - (-j)(-4) = -4 - 4j$$

$$E = 6 + \omega_8^0(10) = 16$$

$$F = -4 + \omega_8^2(-4) = -4 + (-j)(-4) = -4 + 4j$$

$$G = 6 - \omega_8^0(10) = -4$$

$$H = -4 - \omega_8^2(-4) = -4 - (-j)(-4) = -4 - 4j$$

Also.

$$x(0) = A + \omega_8^0 E = 12 + 16$$

$$x(1) = B + \omega_8^1 F = -4 + 4j + (0.707 - 0.707j)(-4 + 4j)$$

$$x(2) = C + \omega_8^2 G = -4 + (-j)(-4)$$

$$x(3) = D + \omega_8^3 H = (-4 - 4j) + (0.707 - 0.707j)(-4 - 4j)$$

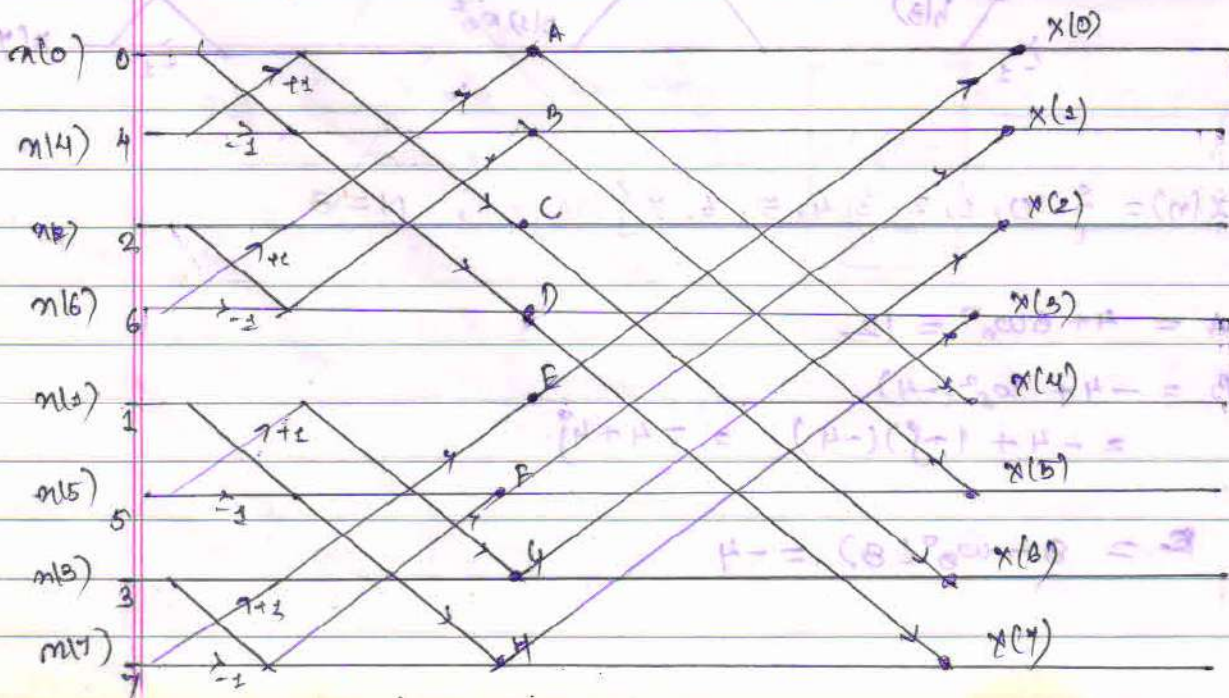
$$x(4) = A - \omega_8^0 E = 12 - 16$$

$$x(5) = B - \omega_8^1 F = -4 - \omega_8^1 F$$

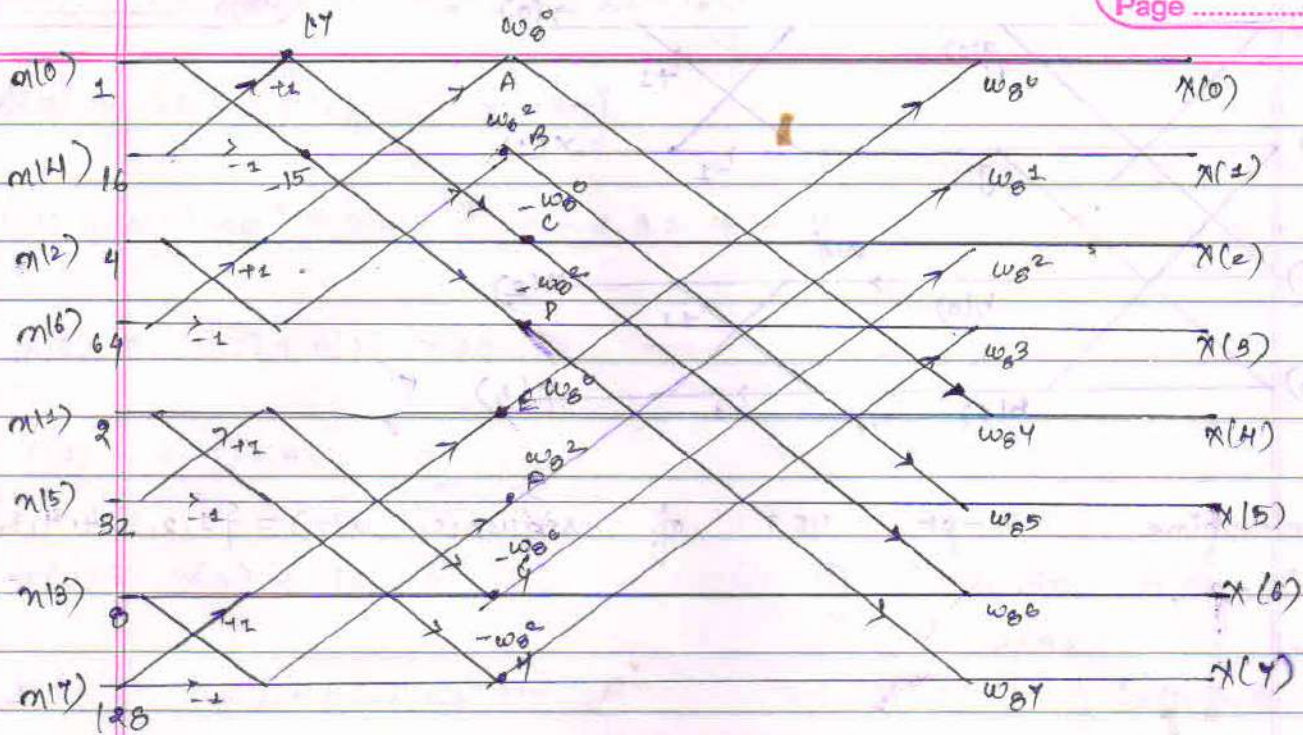
$$x(6) = C - \omega_8^2 G = -4 - (-j)(-4)$$

$$x(7) = D - \omega_8^3 H = -4 - 4j - (\omega_8^3)(-4 - 4j)$$

$$x(k) = \{ 28, -4 + 9.6j, -4 + 4j, -4 + 1.6j, -4, -4 - 1.6j, -4 - 4j, -4 - 9.6j \}$$



Q.  $x(n) = 2^n$  ;  $N = 8$



$A = 17 + \omega_8^0(64) = 81$   
 $B = -15 + \omega_8^2(-60) = -15 + 60j$   
 $C = 17 - \omega_8^0(68) = -51$   
 $D = -15 - \omega_8^2(-60) = -15 - 60j$   
 $E = 34 + \omega_8^0(136) = 170$   
 $F = -30 + \omega_8^2(-120) = -30 + 120j$   
 $G = 34 - \omega_8^0(136) = -102$   
 $H = -30 - \omega_8^2(-120) = -30 - 120j$

Now,  
 $X(0) = A + \omega_8^0 E = 255$   
 $X(2) = B + \omega_8^4 F = 48.63 + j(116.05)$   
 $X(4) = C + \omega_8^2 G = -51 + 102j$   
 $X(6) = D + \omega_8^6 H = -78.63 + 46.05j$   
 $X(1) = A + (-\omega_8^0) E = -85$   
 $X(3) = B - \omega_8^4 F = -78.63 - 46.05j$   
 $X(5) = C - \omega_8^2 G = -51 - 102j$   
 $X(7) = D - \omega_8^6 H = 48.63 - 116.05j$

$\therefore X(k) = \{ 255, 48.63 + 116.05j, -51 + 102j, -78.63 + 46.05j, -85, -78.63 - 46.05j, -51 - 102j, 48.63 - 116.05j \}$

→ ans.