CBCS Course

School of Studies in Statistics

Vikram University, Ujjain

M.A./M.Sc. (Statistics) IV sem

Paper III: Statistical Quality control & Reliability Theory

Title: Property of exponential Distribution:

Suppose n items are under test and the failure time distribution is exponential with mean life θ. The replacement of items that fail by new items makes the test a poission process with intensity

$\lambda = n/\theta$.

Proof: Let N(t) be the number of failures that occured during the interval $[0,t]$. Suppose the Probability of failure during $[t, t+h]$ is Proportional to λ and the interval is so small that the probability of two or more failure in that interval may be neglected, then the process $\{N(t) \mid t \geq 0\}$ is known as poission process with intensity λ.

From (iv) Property it follows that $n x_1/\theta$ is distributed as a gamma variate with parameter 1

i.e. $\frac{n x_1}{\theta} \sim \gamma(1)$

To Prove the property we will use the following property of gamma distribution.

If $x_1, x_2, \ldots x_k$ are i.i.d. as gamma with parameters $n_1, n_2, \ldots n_k$ then $\sum_{i=1}^{k} x_i$ is

also distributed as gamma with parameter $\sum_{i=1}^{k} n_i$

From the above property it follows that

$$n = \sum_{i=1}^{k} \frac{w_i}{\theta} \sim \gamma(k)$$

Since $\frac{n w_i}{\theta} \sim \gamma(1)$

Now,

$$P[N(t) = k] = P[w_1 + w_2 + \cdots + w_k \le t] - P[w_1 + w_2 + \cdots + w_{k+1} \le t]$$

$$= P[w_1 + w_2 + \cdots + w_{k+1} \ge t] - P[w_1 + w_2 + \cdots + w_k \ge t].$$

$$= \frac{1}{\Gamma(k+1)} \int_t^\infty e^{-\frac{ny}{\theta}} \left(\frac{ny}{\theta}\right)^k d\left(\frac{ny}{\theta}\right) - \frac{1}{\Gamma k} \int_t^\infty e^{-\frac{ny}{\theta}} \left(\frac{ny}{\theta}\right)^{k-1} d\left(\frac{ny}{\theta}\right)$$

$$= e^{-\frac{nt}{\theta}} \left[\sum_{r=0}^{k} \left(\frac{nt}{\theta}\right)^r / r! - \sum_{r=0}^{k-1} \left(\frac{nt}{\theta}\right)^r / r! \right]$$

$$= \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad ; \text{ where } \lambda = \frac{n}{\theta}$$

Hence Proved.

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