CBCS course

School of Studies in Statistics
Vikram University, Ujjain

M.A. / M.Sc. IV sem. (Statistics)

Paper III: Statistical Quality Control and Reliability Theory.

Title: Distribution of between failure times (without replacement)

If n items are put to test without replacement and $x_{(1)}, x_{(2)}, \ldots x_{(n)}$ are ordered failure times from an exponential population with mean life θ, then $z_1, z_2 \ldots z_r$ are identically and independently distributed as

$$g(z;\theta) = \frac{1}{\theta} e^{-z/\theta} \quad ; z \geq 0, \theta > 0$$

where

$$z_i = (n-i+1)\{x_{(i)} - x_{(i-1)}\}; \quad i = 1, 2, \ldots n;$$
$$x_{(0)} = 0$$

Since failed items are not replaced, the number of items exposed at $x_{(0)}$ is n, at $x_{(1)}$ is $(n-1)$ -- at $x_{(k)}$ is $(n-k)$ .. and the total time on test upto $x_{(1)}$ is $n x_{(1)}$, upto $x_{(2)}$ is $(n-1)\{x_{(2)} - x_{(1)}\}$ ---, upto $x_{(k)}$ is $(n-k+1)\{x_{(k)} - x_{(k-1)}\}$.

The joint pdf of $X_{(1)}, X_{(2)} --- X_{(n)}$ is

$$g(x_{(1)}, x_{(2)} --- x_{(n)} ; \theta) =$$

$$= \frac{n}{\theta} e^{-n x_1/\theta} \cdot \frac{(n-1)e^{-\frac{(n-1)}{\theta}\{x_{(2)}-x_{(1)}\}}}{\theta} ---- \frac{1}{\theta} e^{-\{x_{(n)}-x_{(n-1)}\}/\theta}$$

$$= \frac{n!}{\theta^n} e^{-\{\sum_{i=1}^{n} x_{(i)}\}/\theta}, \quad 0 < x_{(1)} < x_{(2)} < -- x_{(n)} < \infty$$

Consider the transformation

$$z_1 = n x_{(1)}$$

$$z_2 = (n-1)\{x_{(2)} - x_{(1)}\}$$

$$z_3 = (n-2)\{x_{(3)} - x_{(2)}\}$$

$$\vdots$$

$$z_k = (n-k+1)\{x_{(k)} - x_{(k-1)}\}$$

$$\vdots$$

$$z_n = \{x_{(n)} - x_{(n-1)}\}$$

Now, $$\frac{\partial(z_1, z_2 -- z_n)}{\partial(x_{(1)}, x_{(2)} --- x_{(n)})} = n(n-1) --- 1 = n!$$

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