(i) ${}_nP_x^{(t)}$ = Female Population in the age x to $(x+n)$ at time t

(ii) f_x = Age specific fertility rate in the age x to $(x+n)$, independent of t.

With the above assumptions the component method of population projection leads to the following equation:

$$\frac{1}{2}\left({}_5P_{15}^{(t)} + {}_5P_{15}^{(t+5)}\right)_5 F_{15}\left(\frac{{}_5L_0}{l_0}\right) + \frac{1}{2}\left({}_5P_{20}^{(t)} + {}_5P_{20}^{(t+5)}\right)_5 F_{20}\left(\frac{{}_5L_0}{l_0}\right) + \cdots$$

$$+ \cdots \frac{1}{2}\left({}_5P_{40}^{(t)} + {}_5P_{40}^{(t+5)}\right)_5 F_{40}\left(\frac{{}_5L_0}{l_0}\right) = {}_5P_0^{(t+5)} \quad \text{— (1)}$$

$${}_5P_0^{(t)}\left(\frac{{}_5L_5}{{}_5l_0}\right) = {}_5P_5^{(t+5)} \quad \text{— (2)}$$

$${}_5P_{35}^{(t)}\left(\frac{{}_5L_{40}}{{}_4L_{35}}\right) = {}_5P_{40}^{(t+5)} \quad \text{— (3)}$$

$${}_5P_{40}^{(t)}\left(\frac{{}_5L_{45}}{{}_5L_{40}}\right) = {}_5P_{45}^{(t+5)} \quad \text{— (4)}$$

$$\frac{1}{2}\left({}_5P_{15}^{(t)} + {}_5P_{10}^{(t)}\frac{{}_5L_{15}}{L_{10}}\right){}_5F_{15}\left(\frac{{}_5L_0}{l_0}\right) + \frac{1}{2}\left({}_5P_{20}^{(t)} + {}_5P_{15}^{(t)}\frac{{}_5L_{20}}{{}_5L_{15}}\right){}_5f_{20}\left(\frac{{}_5L_0}{l_0}\right)$$

$$+ \cdots + \frac{1}{2}\left({}_5P_{40}^{(t)} + {}_5P_{35}^{(t)}\frac{{}_5L_{40}}{{}_5L_{35}}\right){}_5f_{40}\left(\frac{{}_5L_0}{l_0}\right) = {}_5P_0^{(t+5)} \quad \text{— (5)}$$

and again (5) can be written as

$$\frac{1}{2}\left(\frac{{}_5L_{15}}{{}_5L_{10}}\,{}_5F_{15}\right)\left(\frac{{}_5L_0}{l_0}\right){}_5P_{10}^{(t)} + \left(f_{15} + \frac{{}_5L_{20}}{{}_5L_{15}}\,{}_5f_{20}\right)\frac{{}_5L_0}{2l_0}\,{}_5P_{15}^{(t)}$$

$$+ \left({}_5F_{20} + \frac{{}_5L_{25}}{{}_5L_{20}}\,{}_5F_{25}\right)\frac{{}_5L_0}{2l_0}\,{}_5P_{20}^{(t)} + \cdots = {}_5P_0^{(t+5)} \quad \text{— (6)}$$

$$\begin{bmatrix} {}_5P_0^{(t+5)} \\ {}_5P_5^{(t+5)} \\ \vdots \\ {}_5P_{45}^{(t+5)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{{}_5L_0}{2l_0}\left(\frac{{}_5L_{15}}{{}_5L_{10}}\,{}_5f_{15}\right) & \frac{{}_5L_0}{2l_0}\left({}_5f_{15} + \frac{{}_5L_{20}}{{}_5L_{15}}\,f_{20}\right) & \cdots & \frac{{}_5L_0}{2l_0}\left({}_5f_{40}\right) \\ \frac{{}_5L_5}{{}_5L_0} & 0 & 0 & 0 & \cdots & 0 \\ 0 & \frac{{}_5L_{10}}{{}_5L_5} & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & 0 & \cdots & \frac{{}_5L_{45}}{{}_5L_{40}} \end{bmatrix}$$

$$\times \begin{bmatrix} {}_5P_0^{(t)} \\ {}_5P_5^{(t)} \\ \vdots \\ {}_5P_{40}^{(t)} \end{bmatrix} \quad \text{— (8)}$$

or $p(t+5) = L\,p(t)$ — (9)

where $p(t+5)$ and $p(t)$ represent the Population age vector in the year $(t+5)$ and t respectively and L is the Leslie matrix. Consisting of elements which are function of fertility and mortality parameters independent of time. This with time independent Leslie matrix as:

$$\left.\begin{aligned} p(t+5) &= L.p(t) \\ p(t+10) &= L.\,p(t+5) = L^2\,p(t) \\ p(t+15) &= L.\,p(t+10) = L^3\,p(t) \\ &\cdots\cdots \end{aligned}\right\} \text{— (10)}$$

For k $p^{(t+5k)} = L^k\,p(t)$

which shows the sequence $p(t)$, $p(t+5)$, $p(t+10)$ - - - Constitute a simple Markov-chain

CBCS Course

S.S. in Statistics
V.U. Ujjain

M.A./M.Sc. II sem (Statistics)

Paper IV: Demography

Title: Leslie Matrix, Markov chain

Teacher: Dr. Ruchi Yadav
9993482294