Some Common Failure Time Distributions (Failure Models) (6)

1. One parameter exponential

$$f(x;\theta) = \frac{1}{\theta} e^{-x/\theta} \quad ; x \geq 0,\ \theta > 0$$

or $$f(x;\lambda) = \lambda e^{-\lambda x} \quad ; x \geq 0,\ \lambda > 0$$

2. Two parameter exponential

$$f(x;\mu,\theta) = \frac{1}{\theta} \exp[-(x-\mu)/\theta] \quad , x \geq \mu,\ \theta > 0.$$

3. Gamma Distribution

$$f(x|\theta, p) = \frac{1}{\Gamma(p)} \frac{x^{p-1}}{\theta^p} \exp(-x/\theta) \quad ; x \geq 0,\ \theta > 0,\ p > 0$$

4. Weibull Distribution

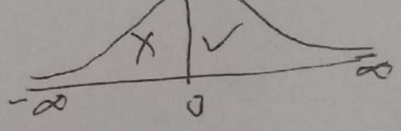
$$f(x;\theta,p) = \frac{p}{\theta} x^{p-1} \exp[-x^p/\theta] \quad ; x \geq 0,\ p > 0,\ \theta > 0.$$

5. Lognormal Distribution

$$f_Y(y) = \frac{1}{\theta\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-\mu}{\theta}\right)^2\right] = \frac{1}{\theta\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu}{\theta}\right)^2\right],$$

where $Y = \ln X$, $X \sim \log n$

6. Truncated Normal Distribution



7. Modified Extreme Value Distribution.

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Mean Time To Failure (MTTF) or Mean Lifetime

Sometimes we are interested in average life time or mean time to failure of an item. This is assumed to be the same for all items which are identical in design and operate under identical conditions. Mean time to failure is the mathematical expectation of the random variable T, denoting average lifetime of an item i.e.

$$MTTF = E[T] = \int_0^\infty t\, f(t)\, dt.$$

Now,

$$f(t) = \frac{d}{dt} F(t) = \frac{d}{dt}[F(t) - 1]$$

$$= -\frac{d}{dt} R(t).$$

Hence.

$$MTTF = -\int_0^\infty t \cdot \frac{d}{dt} R(t)\, dt.$$

$$= -t R(t)\Big|_0^\infty + \int_0^\infty 1 \cdot R(t)\, dt.$$

or $$E[T] = \int_0^\infty R(t)\, dt.$$

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CBCS Course

S. S. in Statistics
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Paper III : Statistical Quality Control & Reliability Theory.

Title : Some common failure time Distribution

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